INFLUENCE OF GROUND PROXIMITY ON AERODYNAMIC FORCES ON AN OSCILLATING TILTED AIRFOIL

W. Fiszdon, W. Lucjanek, and M. Nowak
Polish Academy of Sciences and Technical University
Warsaw, Poland

INTRODUCTION

VTOL or STOL tilted-wing aircraft may, in certain takeoff and landing conditions, operate close to the ground with at least some parts of the wings in a high-velocity stream behind the propellers. The wake behind the wing is deflected by the ground and the vortices shed by the wing then follow a curved path along the streamlines. It was thought that there may be some interest in investigating qualitatively the influence of this deflection on the flutter derivatives to see how much they differ from the derivatives in the absence of ground effect and thus have a certain indication how much they can influence the flutter velocity.

To simplify the analysis, looking for a qualitative estimate a linearized, two-dimensional, incompressible, nonviscous flow was considered and it was assumed that the airfoil is very thin and shed vortices follow the mean streamline which is taken as undisturbed by the presence of the airfoil. The effect of stream boundaries on the nonstationary forces is neglected.

ANALYTICAL FORMULATION OF THE PROBLEM

The geometric configuration and notation is indicated in Fig. 1, where the airfoil semichord b is taken as the unit length.

The normal velocity on the airfoil is assumed to vary according to the following law:

$$w(x)\ e^{i\omega t}$$

where ω is the circular frequency of the airfoil and the positive directions of the velocity components are the same as those of the appropriate coordinate axis.

Possio's integral equation, as shown for example by Garrick [1], giving the relation between the normal velocity and the "reduced" pressure difference is

$$w(x) = k\int_{-1}^{+1} \gamma(\xi)\, K(x,\xi)\, d\xi \tag{1}$$

where $k = \omega b/U$ is the reduced frequency, U the undisturbed flow velocity at infinity, K = kernel given in Eq. (3). The pressure difference on the airfoil is given by:

$$P = -\rho U\gamma(\xi)\, e^{i\omega t} \tag{2}$$

The velocity components parallel to the airfoil surface are taken as constant and equal U.

In the assumed case of incompressible flow $\gamma(\xi)$ is the so-called bound vorticity.

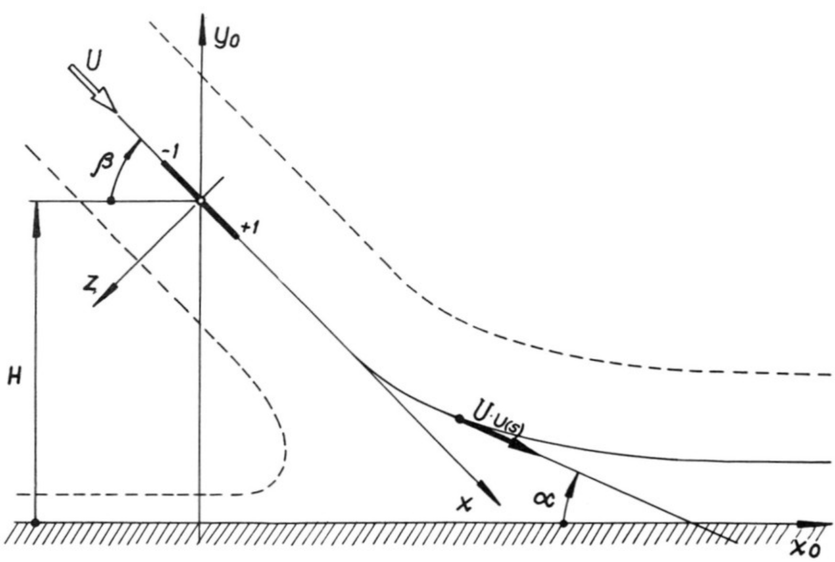


Figure 1.

The kernel of Eq. (1), $K(x,\xi)$ is defined so as to be equal to the induced flow velocity at point x due to a vortex of intensity b/k located at point ξ together with its adjoint free vortices, and it has the following form:

$$K(x,\xi) = \frac{1}{2\pi k}\left[\frac{1}{x-\xi} + f(x,\xi)\right] - \frac{i}{2\pi} e^{ik\xi} \int_\xi^1 e^{-ik\eta} \left[\frac{1}{x-\eta} + f(x,\eta)\right] d\eta + -\frac{i}{2\pi} e^{ik\xi} \int_1^\infty \frac{e^{-ik\tau(s)}}{u(s)} F(x,s)\, ds \tag{3}$$

$\tau(s)$ is defined in Eq. (6). The ground effect, in the above formula, is given by

$$f(x,\xi) = \frac{2(H - x\sin\beta)\sin\beta - (x-\xi)\cos 2\beta}{(x-\xi)^2 + 4(H - x\sin\beta)(H - \xi\sin\beta)} \tag{4}$$

expressing the influence of the ground on the normal velocity component at x induced by a vortex at point ξ, and by:

$$F(x,s) = \frac{[x_0(s) - x\cos\beta]\cos\beta + [H - x\sin\beta + y_0(s)]\sin\beta}{[x_0(s) - x\cos\beta]^2 + [H - x\sin\beta + y_0(s)]^2} + - \frac{[x_0(s) - x\cos\beta]\cos\beta + [H - x\sin\beta - y_0(s)]\sin\beta}{[x_0(s) - x\cos\beta]^2 + [H - x\sin\beta - y_0(s)]^2} \tag{5}$$

expressing the influence of a vortex, located in the wake at a point whose coordinates are $x_0(s)$, $y_0(s)$, on the normal velocity component at the point x of the airfoil.

Calling $Uu(s)$ the velocity of vortex entrainment equal to the velocity on the streamline passing through the trailing edge in steady flow and $t(s)$ the time necessary for a vortex to pass from the center of the airfoil to a point given by the coordinate s measured along the streamline, there exists the following relation between these two magnitudes:

$$\tau(s) = \int_0^s \frac{ds}{u(s)} = \frac{U}{b}\, t(s) \tag{6}$$

In the expression for the kernel, Eq. (3), the first term corresponds to the influence of the bound vortex at point ξ, the second term to the free vortices on the airfoil and the third term to the free vortices in the wake. The first components of the first and second terms are the ones usually encountered in derivative calculations.

For convenience, instead of $t(s)$ we will use the following magnitude, which may be interpreted as a "phase displacement":

$$\zeta(s) = s - \tau(s) = \int_0^s \left(1 - \frac{1}{u(s)}\right) ds \tag{7}$$

and introduce the notation:

$$\delta = k(x - s)$$

and

$$F_1(x,s) = \frac{e^{ik\zeta(s)}}{u(s)} F(x,s) - \frac{1}{x - s} \tag{8}$$

Adding and subtracting from the formula of the kernel, Eq. (3) the following magnitude,

$$\frac{ie^{ik\xi}}{2\pi} \int_1^\infty \frac{e^{-iks}}{x - s} ds \tag{9}$$

the following convenient expression for the kernel is obtained:

$$K(x,\xi) = K_0(\delta) + \frac{1}{2\pi k} \left\{ f(x,\xi) - ike^{ik\xi} \left[\int_\xi^1 e^{-ik\eta} f(x,\eta)\, d\eta + \right.\right.$$
$$\left.\left. - \int_1^\infty F_1(x,s)\, e^{-iks}\, ds \right] \right\} \tag{10}$$

where

$$K_0(\delta) = \frac{1}{2\pi\delta} - \frac{ie^{-i\delta}}{2\pi} \left\{ C_i(\delta) + i\left[Si(\delta) + \frac{\pi}{2} \right] \right\} \tag{11}$$

is the known kernel of Possio's integral equation for an isolated airfoil in the case of incompressible flow [1].

Introducing in Eq. (1) relation (10), the following integral equation for the determination of $\gamma(\xi)$ is obtained:

$$w(x) = K \int_{-1}^{+1} \gamma(\xi)\, K_0(\delta)\, d\xi + \frac{1}{2\pi} \int_{-1}^{+1} \gamma(\xi) \left[f(x,\xi) \right.$$
$$\left. - ike^{ik\xi} \int_\xi^{+1} e^{ik\eta} f(x,\eta)\, d\eta \right] d\xi + - \frac{i}{2\pi} D(x) \int_{-1}^{+1} \gamma(\xi)\, e^{ik\xi}\, d\xi \tag{12}$$

where

$$D(x) = \int_1^\infty F_1(x,s)\, e^{-iks}\, ds \tag{13}$$

This last expression takes into account the modification, of the part of the induced velocity due to the wake, caused by the ground proximity.

ADDITIONAL WAKE INFLUENCE DUE TO GROUND PROXIMITY

The integral in Eq. (13) is a function of x only determined by the geometry of the system in particular by the shape and velocity distribution of the streamline passing through the trailing edge of the airfoil.

Results obtained by Schach [2] were used to determine these last magnitudes.

Denoting $V = ue^{i\alpha}$ and $z = x_0 + iy_0$ the following relation was obtained [2]:

$$\frac{dz}{dv} = \frac{1}{\pi}\left[\frac{1}{v(v-e^{i\beta})} + \frac{1}{v(v-e^{-i\beta})} - \frac{1+\cos\beta}{v(v-1)} - \frac{1-\cos\beta}{v(v+1)}\right] \tag{14}$$

or

$$du + iud\alpha = e^{-i\alpha}[\psi_r(u,\alpha) + i\psi i(u,\alpha)]\, dz \tag{15}$$

where

$$\psi_r = -\frac{\pi}{8\sin^2\beta}[u^4\cos 4\alpha - 2u\cos\beta(u^2\cos 3\alpha - \cos\alpha) - 1] \tag{16}$$

$$\psi_i = -\frac{\pi}{8\sin^2\beta}[u^4\sin 4\alpha - 2u\cos\beta(u^2\sin 3\alpha - \sin\alpha)] \tag{17}$$

Along a stream line

$$dz = e^{-i\alpha}\, ds \tag{18}$$

From Eqs. (15) and (18) the following system of differential equations is obtained:

$$\begin{aligned} \frac{du}{ds} &= \psi_r(u,\alpha)\cos 2\alpha + \psi_i(u,\alpha)\sin 2\alpha \\ \frac{d\alpha}{ds} &= \frac{1}{u}[\psi_i(u,\alpha)\cos 2\alpha - \psi_r(u,\alpha)\sin 2\alpha \end{aligned} \tag{19}$$

The initial values $\alpha(1)$ and $u(1)$ are deduced from the integral of Eq. (15) and relations given in Ref. 2. Knowing $u(s)$ and $\alpha(s)$ the following coordinates of the streamline passing through the trailing edge can be found:

$$\begin{aligned} x_0(s) &= \int_1^s \cos\alpha\, ds + x_0(1) \\ y_0(s) &= -\int_1^s \sin\alpha\, ds + y_0(1) \end{aligned} \tag{20}$$

The system of Eq. (19) was integrated numerically using the 4th order Runge-Kutta method [3], and the integrals of Eqs. (7) and (20), using Simpson's method. Making use of calculated values of $x_0(s)$, $y_0(s)$, $\zeta(s)$ the last integral of Eq. (12) was determined numerically by Filon's [4] method for an upper limit of integration corresponding to

$$x_{ou} \gg H - x \sin\beta$$

As for larger distances $y_0 = y_{0u} \approx$ const, $x_0 \cong s - c$ with c constant, $\zeta(s) = \zeta_u =$ const, $u(s) = 1$ and

$$F(x,s) \approx \frac{2y_{0u}\sin\beta}{(s-c)^2} \tag{21}$$

This part of the integral can be determined in a closed form:

$$\begin{aligned} \int_{s_u}^{\infty} F_1(x,s)\, e^{-iks}\, ds &= 2y_0 \sin\beta e^{-ik(c-\zeta u)} \Bigg\{ k\, (s_i[k(s_u - c)] - \frac{\pi}{2} \\ &+ \Big\{ {}_ic_i[k(s_u-c)]\Big\} + \frac{\theta^{-ik(s_u-c)}}{s_u - c}\Big\} - e^{-ikx}\Big\{ c_i[k\,(s_u - x)] \\ &+ i\left(\frac{\pi}{2} - s_i[k\,(s_u - x)]\right)\Big)\Bigg\} \end{aligned} \tag{22}$$

SOLUTION OF POSSIO'S TYPE EQUATION

Equation (12) was solved by the widely used collocation method [1].

The following series development of the "reduced" pressure difference was applied:

$$\gamma(\xi) = 2U\left[a_0\sqrt{\frac{1-\xi}{1+\xi}} + 2\sqrt{1-\xi^2}\sum_{n=1}^{\infty}\frac{a_n}{n}U_{n-1}(-\xi)\right] \tag{23}$$

where U_n are Tchebyshev's polynomials of the second kind. As is well known, the singularities of the kernel of Eq. (12) can be separated and some of the corresponding integrals calculated in a closed form using known methods—e.g., those given by Frazer [5]. The following results are thus obtained:

$$\int_{-1}^{+1} \frac{\gamma(\xi)}{x - \xi} d\xi = 2\pi U \left[a_0 - 2 \sum_{n=1}^{m} \frac{a_n}{n} T_n(-x) \right]$$

$$\int_{-1}^{+1} \gamma(\xi) \ln |x - \xi| \, d\xi = 2\pi U \left\{ a_0(x - \ln^2) + a_1\left(\frac{1}{2}T_2(x) - \ln^2\right) + \sum_{n=2}^{m} \frac{a_n}{n} \left[\frac{T_n + 1(x)}{n + 1} - \frac{T_{n-1}(-x)}{n - 1} \right] \right\} \tag{24}$$

where m is the number of terms of the series development chosen and T_n are Tchebyshev's polynomials of the first kind.

The remaining integrals are calculated using Gauss's method with $\sqrt{\frac{1 - \xi}{1 + \xi}}$ and $\sqrt{1 - \xi^2}$ as weighting functions (see, for example, Krilov et al., Ref. 6).

From the integral Eq. (12) the following set of linear equations for the complex coefficients a_n of Eq. (23) is obtained:

$$\frac{w(x_i)}{U} = \sum_{n=0}^{m} c_{in} a_n \qquad i = (1, \ldots m + 1) \tag{25}$$

where $m + 1$ is the number of collocation points chosen.

The motions of the airfoil parallel to itself and rotation about its mid-chord point according, respectively, to the relations:

$$h = h_0 e^{i\omega t}$$
$$\alpha = \alpha_0 e^{i\omega t} \tag{26}$$

will be considered. The corresponding amplitude of the velocity of point x of the airfoil is:

$$\frac{w(x)}{U} = \frac{w_n(x)}{U} n_0 + \frac{w_\alpha(x)}{U} \alpha_0 = ikn_0 + (1 + ikx) \alpha_0 \tag{27}$$

and the relevant a_n coefficients are:

$$a_n = a_{nh} h_0 + a_{n\alpha} \alpha_0 \tag{28}$$

From the amplitude of the pressure, Eq. (2), following Garrick [1] the force and moment coefficients deduced are:

$$\begin{aligned} A_{11} &= R_{11} + iI_{11} = 2(a_{0h} + a_{1h}), \\ A_{12} &= R_{12} + iI_{12} = 2(a_{0\alpha} + a_{1\alpha}), \\ A_{21} &= R_{21} + iI_{21} = -\ (a_{0h} + \tfrac{1}{2}a_{2h}), \\ A_{22} &= R_{22} + iI_{22} = -\ (a_{0\alpha} + \tfrac{1}{2}a_{2\alpha}). \end{aligned} \tag{29}$$

The moment coefficients are calculated with reference to the half-chord point.

DISCUSSION OF NUMERICAL RESULTS AND CONCLUSIONS

Numerical calculations were performed on the digital computer GIER.

The width of the air stream was taken equal to the airfoil chord.

Four collocation points were used and their positions were chosen in the zeros of Tchebyshev's polynomials of the first kind.

Sample calculations made for three collocation points have shown that taking them in the above-mentioned points gives much better results at higher frequency coefficients than for an equal distribution of points. Comparing results obtained for a limited number of calculations for 4 and 5 collocation points has shown that the largest differences obtained are below four percent and occurred for the largest altitudes and frequency parameters and are much smaller than one percent for lower parameters.

Calculations were made for the following parameters [7]:

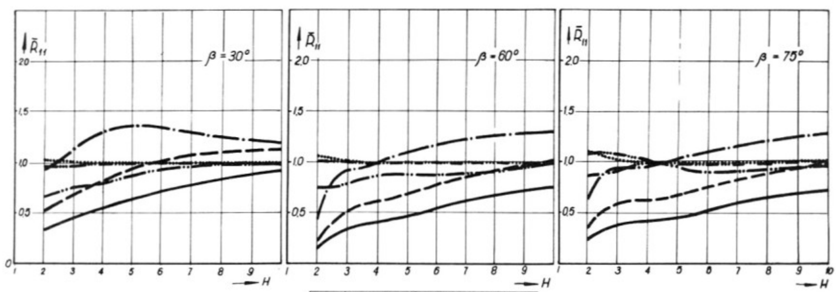
Tilt angle $\beta = 30°, 45°, 60°, 75°$
Height $H = 2, 3, 4, 6, 10$
Frequency parameter $k = 0.05, 0.1, 0.2, 0.5, 1.0, 1.5, 2.0, 2.5$.

It is noted that for heights below two chord lengths ($H < 4$) measured from the midchord point, the results are less accurate, because of the large deviations from the simplifying assumptions made, concerning the stream line passing through the trailing edge and the velocity along the airfoil.

Comparative calculations were made for different simplifications concerning the shape of stream line passing through the trailing edge which was taken as straight with constant velocity distribution and the same velocity distribution along the calculated curve stream line. The results obtained show that the coefficients R_{12}, R_{22}, I_{11}, I_{21} give the same results independently of the above given assumptions. The other coefficients indicate large variations for altitudes $H < 6$ and small angles of tilt and $H < 10$ for large tilt angles.

The main results obtained are shown on the curves given in Figs. 2–5.

The direct force coefficients R_{11} for $k > 1$ are nearly equal to the values called classical, e.g., for $H = \infty$, this due to the predominating influence of the adjoint mass at high frequencies; at lower values of the frequencies the effect of the wake is most important and therefore large relative variations of the small coefficients occur.

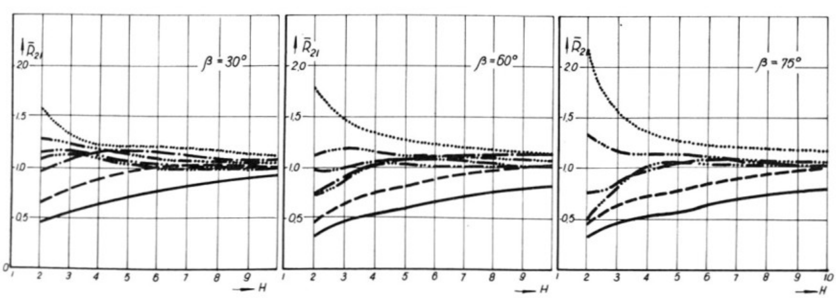


$$\bar{R}_{11} = \frac{R_{11}}{R_{11}(cl)}$$

k	type of line used	$R_{11}(cl)$, value of coefficient for $H = \infty$
0.05	———	+ 0.0106
0.1	— — —	+ 0.0245
0.2	—·—	+ 0.0354
0.5	—··—	− 0.0993
1.0	—···—	− 0.7994
2.5	··········	− 6.0135

Note: As for k close to 0.34 the value of $R_{11}(cl)$ becomes zero the corresponding $\bar{R}_{11}$ values tend to infinity.

Figure 2.



$$\bar{R}_{21} = \frac{R_{21}}{R_{21}(cl)}$$

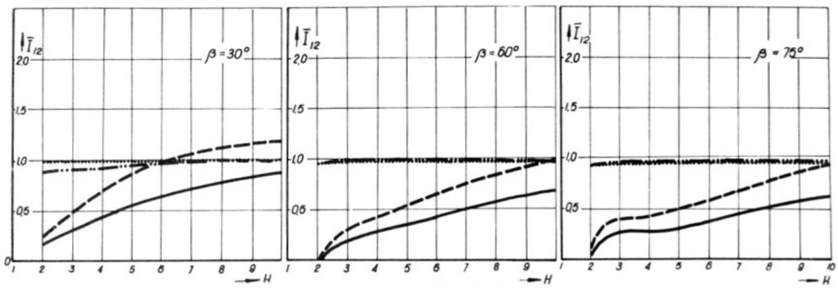
k	type of line used	$R_{21}(cl)$, value of coefficient for $H = \infty$
0.05	———	− 0.0065
0.1	— — —	− 0.0172
0.2	—·—	− 0.0377
0.5	—··—	− 0.0753
1.0	—···—	− 0.1003
2.0	—····—	− 0.1154
2.5	··········	− 0.1182

Figure 3.

The coupling R_{21} coefficient varies considerably within the whole range of parameter variations considered and these coefficients are smaller from their classical values for small frequency coefficients and larger when these values are larger.

The damping coefficients I_{12} and I_{22} vary appreciably for smaller values of the frequency coefficients and behave similarly to R_{11}.

The remaining coefficients show relatively small differences as compared with their classical values and these are below a few percent for $H \geqslant 4$.



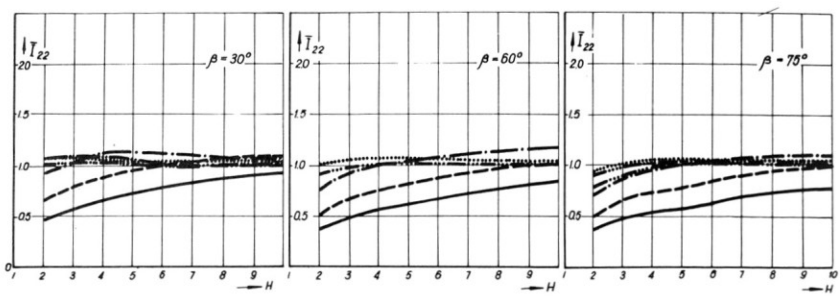
$$\bar{I}_{12} = \frac{I_{12}}{I_{12}(cl)}$$

k	type of line used	$I_{12}(cl)$, value of coefficient for $H = \infty$
0.05	———	− 0.1658
0.1	— — —	− 0.1614
0.5	—··—	+ 0.4976
1.0	—···—	− 1.3389
2.5	··········	− 3.6773

Note: As for k close to 0.22 the value of $I_{12}(cl)$ becomes zero the corresponding $\bar{I}_{12}$ values tend to infinity.

Fig. 4

Figure 4.



$$\bar{I}_{22} = \frac{I_{22}}{I_{22}(cl)}$$

k	type of line used	$I_{22}(cl)$, value of coefficient for $H = \infty$
0.05	———	+ 0.1329
0.1	— — —	+ 0.1807
0.2	—·—	+ 0.2159
0.5	—··—	+ 0.2512
1.0	—···—	+ 0.3306
2.0	—····—	+ 0.5447
2.5	··········	+ 0.6614

Fig. 5

Figure 5.

It is impossible to make at present any definite statements concerning the effect of the ground proximity on the critical flutter velocity of a tilted wing. However, as the relatively small coupling moment and force coefficients R_{21} and I_{12} change considerably, their influence on flutter, particularly for frequency coefficients $k < 1$, should be carefully analyzed at least for a few typical cases.

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